A Dynamical Brane in the Gravitational Dual of $\mathcal{N}=2$ $Sp(N)$ Superconformal Field Theory

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The particular model of d5 higher derivative gravity which is dual to $\mathcal{N}=2$ $Sp(N)$ SCFT is considered. A (perturbative) AdS black hole in such theory is constructed in the next-to-leading order of the AdS/CFT correspondence. The surface counterterms are fixed by the conditions required for a well-defined variational procedure and the finiteness of AdS space (when the brane goes to infinity). A dynamical brane is realized at the boundary of an AdS black hole with a radius that is larger than the horizon radius. The AdS/CFT correspondence dictates the parameters of the gravitational dual in such a way that the dynamical brane (the observable universe) always occurs outside the horizon.

§1. Introduction

In modern studies of brane-worlds, one assumes that the observable universe lies as a boundary in multi-dimensional bulk space, where gravity on the brane is trapped [1], and bulk represents some AdS-like multidimensional background which follows from (IIB) string theory. The brane-world cosmology [2,3] (and references therein) is usually constructed from the (d5) AdS bulk space with the following conditions:

a. There are few free parameters of the that must be fine-tuned to get the desirable properties. In most cases these parameters are the (negative) bulk cosmological constant and brane cosmological constant (brane tension). Playing with these two parameters one has already constructed various brane-worlds in Einstein gravity (with surface terms). The coefficients of higher derivative terms in the bulk (or brane) action may play the role of these parameters as well.

b. As the bulk, one chooses AdS space or its product with some other manifold.

Clearly, the above mechanism for the realization of the observable brane universe is not dynamical, as the parameters of the theory are fine-tuned to get brane-world. Presumably, a dynamically generated brane should be searched for within the AdS/CFT correspondence [4], where a warped compactification introduces the

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brane-world set-up. Indeed, one version of such a scenario that is more suitable in the framework of the AdS/CFT correspondence has been presented in Refs.5), where the brane tension is fixed from the beginning, but the quantum CFT existing on the brane produces an effective brane tension. This scenario is much more restrictive because brane-worlds are produced completely dynamically. Our approach follows this line.

From another point of view, it is not clear what the bulk should be. It is expected that the topology [6)] should be very important in realization of warped compactification in string theory in frames of the AdS/CFT correspondence. In a recent paper [7)] we considered the particular model of d5 higher derivative gravity that possesses a Schwarzschild-anti de Sitter (S-AdS) black hole as an exact bulk solution. It has been shown that a four-dimensional brane (flat or de Sitter) could be realized dynamically. Hence, the observable universe may represent the boundary of a higher dimensional AdS black hole.

In the present work we generalize the mechanism of Ref.7) and show that it may be realized completely within the AdS/CFT correspondence. The starting point is the particular model of d5 higher derivative (HD) gravity that appears as the low-energy limit of compactified IIB string theory. All parameters of the model are defined. The theory is expected to be dual to $\mathcal{N}=2$ SCFT with the gauge group $Sp(N)$ [8)] (see also Ref.9)). This duality has also been checked by comparison of holographic and QFT conformal anomalies in Refs.10) and 11) in the next-to-leading order of the large-N expansion.

The surface counterterms are added to the starting action in frames of the AdS/CFT correspondence. The parameters of these counterterms are not arbitrary. They are fixed by two conditions:

a. The variational procedure should be well-defined.

b. The leading divergences of a bulk AdS black hole should cancell (finiteness).

In particular, the brane tension is fixed by these conditions.

Working with such a gravitational dual of $\mathcal{N}=2$ SCFT, we construct a AdS black hole perturbatively in the next-to-leading order of the AdS/CFT correspondence. The (flat) brane is dynamically created as the boundary of such an AdS black hole and its radius, which is larger than horizon radius, is found. An analogous AdS-like cosmological model is also briefly discussed.

§2. AdS black hole with dynamical brane

Let us start from the general action of higher derivative gravity. It is given by *)

$$S=\int d^{d+1}x\sqrt{-\hat{G}}\left\{a\hat{R}^2+b\hat{R}_{\mu\nu}\hat{R}^{\mu\nu}+c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}+\frac{1}{\kappa^2}\hat{R}-\Lambda\right\}\ . \qquad (2\cdot 1)$$

The equations of motion derived from the above action (2·1) are

$$\begin{aligned}0=&-\frac{1}{2}\hat{G}_{\zeta\xi}\left\{a\hat{R}^2+b\hat{R}_{\mu\nu}\hat{R}^{\mu\nu}+c\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma}+\frac{1}{\kappa^2}\hat{R}-\Lambda\right\}\\&+2a\hat{R}\hat{R}_{\zeta\xi}+2b\hat{R}_{\mu\zeta}\hat{R}^{\mu}_{\xi}+2c\hat{R}_{\zeta\mu\nu\rho}\hat{R}^{\mu\nu\rho}_{\xi}+\frac{1}{\kappa^2}\hat{R}_{\zeta\xi}\\&-a\left(D_\zeta D_\xi+D_\xi D_\zeta\right)\hat{R}\\&+b\left(\hat{G}_{\zeta\xi}D_\rho D_\sigma\hat{R}^{\rho\sigma}-D_\xi D_\sigma\hat{R}^{\sigma}_{\zeta}-D_\zeta D_\sigma\hat{R}^{\sigma}_{\xi}+\Box\hat{R}_{\xi\zeta}\right)+4cD_\rho D_\kappa\hat{R}_{\zeta}{}^{\rho}{}_{\xi}{}^{\zeta}\end{aligned}\qquad (2\cdot 2)$$

We choose the metric

$$ds^2=\hat{G_{\mu\nu}}dx^\mu dx^\nu=-\mathrm{e}^{2\rho}dt^2+\mathrm{e}^{-2\rho}dr^2+r^2\sum_{i=1}^{d-1}\left(dx^i\right)^2\ . \qquad (2\cdot 3)$$

Here the 4d part of the metric is chosen to be flat only for simplicity. There is no problem in considering the de Sitter or anti-de Sitter 4d part of the metric. (The only difference in this case is that the corresponding calculations are technically a bit more involved.)

If we further assume a, b and c are small compared with $\frac{1}{\kappa^2}$, we obtain the following AdS black hole solution when $d+1=5$:

$$\mathrm{e}^{2\rho}=\frac{1}{r^2}\left\{-\mu+\left(1+\frac{20a\kappa^2}{3}+\frac{4b\kappa^2}{3}+\frac{2c\kappa^2}{3}\right)r^4+\frac{2c\kappa^2\mu^2}{r^4}\right\}\ . \qquad (2\cdot 4)$$

As the leading-order effect of a and b is simply to change the radius of AdS, we only consider the case $a=b=0$ in the following (only Riemann curvature squared terms):

$$S=\int d^{d+1}x\sqrt{-\hat{G}}\left\{\frac{1}{\kappa^2}\left(\hat{R}+12\right)+c\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}\right\}. \qquad (2\cdot 5)$$

In particular, when $d+1=5$ and

$$\frac{1}{\kappa^2}=\frac{N^2}{4\pi^2}\ ,\quad c=\frac{6N}{24\cdot 16\pi^2}\ , \qquad (2\cdot 6)$$

*) The conventions for curvatures are defined by

$$R=g^{\mu\nu}R_{\mu\nu},\quad R_{\mu\nu}=-\Gamma^{\lambda}_{\mu\lambda,\nu}+\Gamma^{\lambda}_{\mu\nu,\lambda}-\Gamma^{\eta}_{\mu\lambda}\Gamma^{\lambda}_{\nu\eta}+\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\lambda\eta}\ ,$$

$$R^{\lambda}_{\mu\nu\kappa}=\Gamma^{\lambda}_{\mu\kappa,\nu}-\Gamma^{\lambda}_{\mu\nu,\kappa}+\Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta}-\Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta},\quad \Gamma^{\eta}_{\mu\lambda}=\frac{1}{2}g^{\eta\nu}\left(g_{\mu\nu,\lambda}+g_{\lambda\nu,\mu}-g_{\mu\lambda,\nu}\right)\ .$$

the action (2·5) appears as the low-energy theory of type IIB string theory on $AdS_5\times X_5$ where $X_5 = S^5/Z_2$ 8), 9). This theory is believed to be dual to $\mathcal{N}=2$ theory with the gauge group $Sp(N)$.

When $a=b=0$, the perturbative solution in (2·4) looks like [12)]

$$\mathrm{e}^{2\rho} = \frac{1}{r^2}\left\{-\mu + \left(1+\frac{2}{3}\epsilon\right)r^4 + 2\epsilon\frac{\mu^2}{r^4}\right\}, \quad \epsilon \equiv c\kappa^2 \ . \tag{2·7}$$

In the following, we ignore the terms containing higher powers of ϵ. If we assume the metric to take the form in (2·3), the components of the Ricci tensor and Riemann tensor are given by

$$\begin{aligned}
\hat{R}_{tt} &= \left(\rho'' + 2(\rho')^2 + \frac{(d-1)\rho'}{r}\right)\mathrm{e}^{4\rho} \ , \\
\hat{R}_{rr} &= -\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r} \ , \\
\hat{R}_{ij} &= \left(-2r\rho' - d + 2\right)\mathrm{e}^{2\rho}\delta_{ij} \ , \\
\text{other Ricci tensor components } &= 0 \ ,
\end{aligned} \tag{2·8}$$

$$\begin{aligned}
\hat{R}_{trtr} &= -\hat{R}_{trrt} = -\hat{R}_{rttr} = \hat{R}_{rtrt} \ , \\
&= \mathrm{e}^{2\rho}\left(\rho'' + 2\rho'^2\right) \\
\hat{R}_{titj} &= -\hat{R}_{ittj} = -\hat{R}_{tijt} = \hat{R}_{itjt} \ , \\
&= r\rho'\delta_{ij}\mathrm{e}^{4\rho} \ , \\
\hat{R}_{rirj} &= -\hat{R}_{rijr} = -\hat{R}_{irrj} = \hat{R}_{irjr} \ , \\
&= -r\rho'\delta_{ij} \ , \\
\hat{R}_{ijkl} &= -r^2\mathrm{e}^{2\rho}\left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}\right) \ , \\
\text{other Riemann tensor components } &= 0 \ .
\end{aligned} \tag{2·9}$$

The scalar curvature and the square of the Riemann tensor are

$$\begin{aligned}
\hat{R} &= \left(-2\rho'' - 4(\rho')^2 - \frac{4(d-1)\rho'}{r} - \frac{(d-2)(d-1)}{r^2}\right)\mathrm{e}^{2\rho} \\
\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} &= 4\hat{R}_{trtr}\hat{R}^{trtr} + 4\hat{R}_{titj}\hat{R}^{titj} + 4\hat{R}_{rirj}\hat{R}^{rirj} + \hat{R}_{ijkl}\hat{R}^{ijkl} \\
&= 4\mathrm{e}^{4\rho}\left(\rho'' + 2\rho'^2\right)^2 + 4(d-1)r^{-2}\rho'^2\mathrm{e}^{4\rho} + 4(d-1)r^{-2}\rho'^2\mathrm{e}^{4\rho} \\
&\quad +2(d-1)(d-2)r^{-4}\mathrm{e}^{4\rho} \\
&= 4\mathrm{e}^{4\rho}\left(\rho'' + 2\rho'^2\right)^2 + 8(d-1)r^{-2}\rho'^2\mathrm{e}^{4\rho} + 2(d-1)(d-2)r^{-4}\mathrm{e}^{4\rho}.
\end{aligned} \tag{2·10}$$

We assume that there is a boundary at $r=r_0$, where the brane lies. Then we need to add a boundary term, specifically, a 4 dimensional cosmological term, in order to realize brane-world universe, i.e. the RS scenario for 4d gravity. [1)] Usually such a surface term is chosen to be arbitrary. Its fine-tuning is responsible for the creation of the brane-world.

In our scenario (within the AdS/CFT correspondence), the surface counterterms are not arbitrary. Such a surface term causes the variational principle to be well-defined and a complete AdS space to be finite when the brane goes to infinity. We take the surface terms in the following form: [13)]

$$\begin{aligned}
S_b &= S_b^{(1)} + S_b^{(2)} \qquad (2\cdot11)\\
S_b^{(1)} &= \int d^4x\sqrt{\hat{g}}\Big[4\tilde{a}\hat{R}D_\mu n^\mu + 2\tilde{b_1}n_\mu n_\nu \hat{R}^{\mu\nu}D_\sigma n^\sigma + 2\tilde{b_2}\hat{R}_{\mu\nu}D^\mu n^\nu \\
&\qquad +8\tilde{c}n_\mu n_\nu \hat{R}^{\mu\tau\nu\sigma}D_\tau n_\sigma + \frac{2}{\tilde{\kappa}^2}D_\mu n^\mu\Big] ,\\
S_b^{(2)} &= -\eta\int d^4x\sqrt{\hat{g}} ,
\end{aligned}$$

where the normal vector n^μ and its covariant derivatives are given by

$$\begin{aligned}
&n^r = \mathrm{e}^\rho, \quad \text{other components} = 0\\
&D_r n^r = 0, \quad D_t n^t = \mathrm{e}^\rho\rho', \quad D_i n^j = \frac{\mathrm{e}^\rho}{r}\delta_i^j\\
&D_\mu n^\mu = \mathrm{e}^\rho\rho' + \frac{(d-1)\mathrm{e}^\rho}{r} . \qquad (2\cdot12)
\end{aligned}$$

In (2·11), we can choose $\tilde{b}_1 = \tilde{b}_2$, but as we see below, it is convienient to treat them as independent parameters when we considers the black hole background as in Ref.7).

When we substitute the solution (2·7) into the bulk action (2·5) with $d = 4$, there appears a divergence if there is no brane, which can be a boundary of the spacetime. In order to regularize the divergence, we restrict the integration over r to be a finite region ($\int d^5x \to \int d^4x \int_0^r dr$) and we assume the surface terms in (2·11) appear on the boundary. The parameter η in (2·11) is determined by the condition that the corresponding term cancel the leading divergence. After the integration over r, we find

$$S \sim \frac{r^4}{\kappa^2}\left(-2+\frac{20}{3}\epsilon\right)\int d^4x + o\left(r^4\right) , \qquad (2\cdot13)$$

and the surface terms in (2·11) behave, when r is large, as follows:

$$\begin{aligned}
S_b \sim r^4 \Big\{&-320\tilde{a} - 32\tilde{b}_1 - 32\tilde{b}_2 - 32\tilde{c} + \frac{8}{\tilde{\kappa}^2}\left(1+\frac{2}{3}\epsilon\right)\\
&-\eta\left(1+\frac{1}{3}\epsilon\right)\Big\}\int d^4x + o\left(r^4\right) . \qquad (2\cdot14)
\end{aligned}$$

Then we obtain

$$\eta = \frac{1}{\kappa^2}\left(-2+\frac{22}{3}\epsilon\right) - 320\tilde{a} - 32\tilde{b}_1 - 32\tilde{b}_2 - 32\tilde{c} + \frac{8}{\tilde{\kappa}^2}\left(1+\frac{1}{3}\epsilon\right) . \qquad (2\cdot15)$$

The variation of the action (2·5) and (2·11) on the boundary, which lies at $r = r_0$, gives

$$\delta S|_{r=r_0} = \int d^4x r_0^{d-1}\mathrm{e}^{2\rho}$$

$$\times \left[\frac{1}{\kappa^2} \left\{ -2\delta\rho' + \delta\rho \left(-8\rho' - \frac{4(d-1)}{r_0} \right) \right\} \right.$$
$$\left. + c\mathrm{e}^{2\rho} \left\{ 8(\rho'' + 2(\rho')^2)(\delta\rho' + 4\rho'\delta\rho) + \frac{16(d-1)}{r_0^2}\rho'\delta\rho \right\} \right] , \qquad (2\cdot16)$$

$$\delta S_b = \int d^4x r_0^{d-1} \mathrm{e}^{\rho}$$
$$\times \left[\mathrm{e}^{3\rho}\delta\rho'' \left\{ \left(-8\tilde{a} - 2\tilde{b}_1 \right) \left(\rho' + \frac{d-1}{r_0} \right) - 2\tilde{b}_2\rho' - 8\tilde{c}\rho' \right\} \right.$$
$$+\delta\rho' \left\{ \tilde{a} \left\{ \left(-32\rho' - \frac{16(d-1)}{r_0} \right) \left(\rho' + \frac{d-1}{r_0} \right) \mathrm{e}^{3\rho} \right. \right.$$
$$+4 \left(\left(-2\rho'' - 4(\rho')^2 - \frac{4(d-1)\rho'}{r_0} - \frac{(d-2)(d-1)}{r_0^2} \right) \mathrm{e}^{2\rho} \right.$$
$$\left. \left. + \frac{(d-1)k}{r_0^2} \right) \mathrm{e}^{\rho} \right\}$$
$$+\tilde{b}_1 \left\{ \left(-8\rho' - \frac{2(d-1)}{r_0} \right) \left(\rho' + \frac{d-1}{r_0} \right) \mathrm{e}^{3\rho} \right.$$
$$\left. +2 \left(-\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r_0} \right) \mathrm{e}^{3\rho} \right\}$$
$$+\tilde{b}_2 \left\{ 2 \left(-\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r_0} \right) \mathrm{e}^{3\rho} \right.$$
$$\left. -8(\rho')^2 \mathrm{e}^{3\rho} - \mathrm{e}^{3\rho}\rho' \frac{2(d-1)}{r_0} - \mathrm{e}^{3\rho} \frac{4(d-1)}{r_0^2} \right\} + \frac{2\mathrm{e}^{\rho}}{\tilde{\kappa}^2}$$
$$\left. +8\tilde{c}\mathrm{e}^{3\rho} \left(-6(\rho')^2 - \rho'' - \frac{(d-1)}{r_0^2} \right) \right\}$$
$$+\delta\rho \left\{ 4\tilde{a} \left\{ 4\mathrm{e}^{3\rho} \left(-2\rho'' - 4(\rho')^2 - \frac{4(d-1)\rho'}{r_0} \right. \right. \right.$$
$$\left. \left. - \frac{(d-2)(d-1)}{r_0^2} \right) \left(\rho' + \frac{d-1}{r_0} \right) + \frac{2\mathrm{e}^{\rho}(d-1)k}{r_0^2} \left(\rho' + \frac{d-1}{r_0} \right) \right\}$$
$$+8\tilde{b}_1 \left(-\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r_0} \right) \left(\rho' + \frac{d-1}{r_0} \right) \mathrm{e}^{3\rho}$$
$$+2\tilde{b}_2 \left\{ 4\rho' \left(-\rho'' - 2(\rho')^2 - \frac{(d-1)\rho'}{r_0} \right) \mathrm{e}^{3\rho} \right.$$
$$+ \frac{4(d-1)}{r_0^3} \left(-2r\rho' - d + 2 \right) \mathrm{e}^{3\rho}$$
$$\left. + \frac{2k(d-1)}{r^3} \right\} + \frac{4\mathrm{e}^{\rho}}{\tilde{\kappa}^2} \left(\rho' + \frac{d-1}{r_0} \right)$$
$$\left. \left. -32\tilde{c}\mathrm{e}^{3\rho} \left((\rho'' + 2\rho'^2)\rho' + \frac{\rho'(d-1)}{r_0^2} \right) - \eta \right\} \right] . \qquad (2\cdot17)$$

To satisfy the condition that the variational principle in the theory under discussion be well-defined, the coefficients of $\delta\rho''$ and $\delta\rho'$ must vanish. From the condition that

the coefficient of $\delta\rho''$ vanishes, we obtain

$$\tilde{b}_1 = -4\tilde{a}, \quad \tilde{b}_2 = -4\tilde{c} \ . \tag{2·18}$$

Then, substituting the solution (2·7) and the condition (2·18), we find (putting $d=4$)

$$\begin{aligned}
&\delta S + \delta S_b \\
&= \int d^4x r_0^3 \Big[\delta\rho' \Big(2\left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right) + 8c\left(-3r_0^{-4}\mu + 1\right) \\
&\quad -12\tilde{a}(8r_0^{-4}\mu + 12(1-\mu r_0^{-4})) \\
&\quad +24\tilde{c}(3 + r_0^{-4}\mu) \Big) \mathrm{e}^{2\rho} \\
&\quad +\delta\rho \Big(-\frac{4}{\kappa^2}\left\{ -\mu r_0^{-3} + 5\left(1+\frac{2}{3}\epsilon\right) r_0 - 6\epsilon\mu^2 r_0^{-7} \right\} \\
&\quad +\frac{4}{\tilde{\kappa}^2}\left\{ -2r_0^{-3}\mu + 4\left(1+\frac{2}{3}\epsilon\right) r_0 \right\} \\
&\quad +16c\left\{ -9\mu^2 r_0^{-7} - 4r_0^{-3}\mu + 5r_0 \right\} \\
&\quad -48\tilde{a}\left\{ -8r_0^{-3}\mu + 16r_0 \right\} \\
&\quad +96\tilde{c}\left\{ 2\mu^2 r_0^{-7} - 2\mu r_0^{-3} + 4r_0 \right\} \\
&\quad -\eta \frac{\sqrt{-\mu + r_0^4}}{r_0} \left\{ 1 + \frac{\epsilon r_0^4}{3\left(-\mu + r_0^4\right)} - \frac{\epsilon\mu^2}{r_0^4\left(-\mu + r_0^4\right)} \right\} \Big) \Big] \ .
\end{aligned} \tag{2·19}$$

The condition that the coefficient of $\delta\rho'$ vanishes gives $\tilde{a}$ and $\tilde{c}$ as

$$\begin{aligned}
\tilde{a} &= \frac{1}{144}\left(40c + \left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right)\right) \ , \\
\tilde{c} &= \frac{4}{9}c - \frac{1}{72}\left(\frac{1}{\tilde{\kappa}^2} - \frac{1}{\kappa^2}\right) \ .
\end{aligned} \tag{2·20}$$

Using the above expressions for $\tilde{a}$ and $\tilde{c}$, and as well as the relation $c = \frac{\epsilon}{\kappa^2}$, the condition that the coefficient of $\delta\rho$ vanishes leads to

$$\begin{aligned}
0 &= F(r_0) \\
&\equiv \frac{1}{\tilde{\kappa}^2}\left\{ -\frac{8}{3}\mu^2 r_0^{-7} - \frac{8}{3}\mu r_0^{-3} + \frac{16}{3}(1+2\epsilon) r_0 \right\} \\
&\quad +\frac{1}{\kappa^2}\left\{ -\frac{8}{3}(13\epsilon - 1)\mu^2 r_0^{-7} - \frac{4}{3}(32\epsilon + 1)\mu r_0^{-3} + \left(24\epsilon - \frac{28}{3}\right) r_0 \right\} \\
&\quad -\eta \frac{\sqrt{-\mu + r_0^4}}{r_0} \left\{ 1 + \frac{\epsilon r_0^4}{3\left(-\mu + r_0^4\right)} - \frac{\epsilon\mu^2}{r_0^4\left(-\mu + r_0^4\right)} \right\} \ .
\end{aligned} \tag{2·21}$$

Using (2·15), (2·18) and (2·20), we find that η in (2·21) has the following form:

$$\eta = \frac{2}{3\kappa^2}(1 - 5\epsilon) + \frac{16}{3\tilde{\kappa}^2}\left(1 + \frac{1}{2}\epsilon\right) \ . \tag{2·22}$$

Hence, the coefficients of the surface counterterms in the AdS/CFT correspondence are now fixed.

Equation (2·21) can be regarded as the equation to determine r_0, i.e., the position of the brane. When $r_0 \to \infty$, $F(r_0)$ behaves as

$$\begin{aligned} F(r_0) &\sim \left\{ \frac{1}{\tilde{\kappa}^2}\left(\frac{16}{3} + \frac{32}{3}\epsilon\right) + \frac{1}{\kappa^2}\left(-\frac{28}{3} + 24\epsilon\right) - \eta\left(1 + \frac{\epsilon}{3}\right)\right\} r_0 \\ &\sim -\frac{10}{\kappa^2} + \mathcal{O}(\epsilon) < 0 \ . \end{aligned} \tag{2·23}$$

On the other hand, when $r_0 \to \mu^{\frac{1}{4}}$, $F(r_0)$ behaves as

$$F(r_0) \sim \frac{2}{3}\frac{\eta\epsilon\mu}{\sqrt{-\mu + r_0^4}} \ . \tag{2·24}$$

In case of the string theory dual to the $\mathcal{N} = 2$ theory with the gauge group $Sp(N)$ in (2·6), c and, therefore, ϵ are positive. Combining (2·23) and (2·24), we find that there is a solution r_0 satisfying the brane equation (2·21) in the $\mathcal{N} = 2$ SCFT case. Equations (2·23) and (2·24) imply that $r_0^4 - \mu = \mathcal{O}\left(\epsilon^2\right)$. Then, assuming

$$r_0^4 = \mu + \alpha^2\epsilon^2 + \mathcal{O}\left(\epsilon^3\right) \quad (\alpha > 0) \tag{2·25}$$

and substituting (2·22) and (2·25) into (2·21), we find

$$\alpha = \frac{2}{3}\cdot\frac{1 + \frac{\tilde{\kappa}^2}{8\kappa^2}}{1 + \frac{13\tilde{\kappa}^2}{8\kappa^2}} + \mathcal{O}\left(\epsilon\right) \ . \tag{2·26}$$

In particular, if we choose $\tilde{\kappa}^2 = \kappa^2$, as in the original Gibbons-Hawking term,[14] we obtain

$$\alpha = \frac{2}{7} \ , \tag{2·27}$$

which yields

$$r_0^4 = \mu + \left(\frac{2}{7}\epsilon\right)^2 + \mathcal{O}\left(\epsilon^3\right) \ . \tag{2·28}$$

We should note that the solution for r_0 gioven by (2·26) [with (2·25)] or (2·28) represents a larger value than the unperturbative horizon which lies at $r = \mu^{\frac{1}{4}}$ (in terms of mass of the AdS black hole under consideration). As we see, the c correction makes the radius of the horizon smaller if c or ϵ is positive. Then, the brane always exists outside the horizon. In other words, AdS/CFT duality predicts the correct signs of the gravitational action in such a way that the observable universe is realized as the brane outside the multi-dimensional black hole horizon. The whole evolution of theuniverse could occur within less than one period of the black hole time.

Let us consider the thermodynamic quantities. In the solution given by (2·7), the radius r_h of the horizon and the temperature T are given by

$$r_h \equiv \mu^{\frac{1}{4}}\left(1 - \frac{2}{3}\epsilon\right) \ , \quad T = \frac{\mu^{\frac{1}{4}}}{\pi}\left(1 - 2\epsilon\right) = \frac{\mu^{\frac{1}{4}}}{\pi}\left(1 - \frac{1}{8N}\right) \ . \tag{2·29}$$

After Wick-rotating the time variables by $t \to i\tau$, the free energy $\mathcal{F}$ can be obtained from the action S in (2·1) with $a = b = 0$, where the classical solution is substituted. We find

$$\mathcal{F} = \frac{1}{T} S \; . \tag{2·30}$$

Using (2·2) with $a = b = 0$, (2·6) and (2·7), we find

$$\begin{aligned} S &= \frac{N^2}{4\pi^2} \int d^5x \sqrt{g} \left\{ 8 - \frac{2\epsilon}{3} \left(40 + \frac{72\mu^2}{r^8} \right) \right\} \\ &= \frac{N^2 V_3}{4\pi^2 T} \int_{r_h}^{\infty} dr r^3 \left\{ 8 - \frac{2\epsilon}{3} \left(40 + \frac{72\mu^2}{r^8} \right) \right\} \; . \end{aligned} \tag{2·31}$$

Here V_3 is the volume of 3d flat space, and we have assumed that τ has a period of $\frac{1}{T}$. The expression of S contains the divergence coming from large r. In order to subtract the divergence, we regularize S in (2·31) by cutting off the integral at a large radius $r_{\max}$ and subtracting the solution with $\mu = 0$. This yields

$$\begin{aligned} S_{\text{reg}} = \frac{N^2 V_3}{4\pi^2 T} &\left(\int_{r_h}^{\infty} dr r^3 \left\{ 8 - \frac{2\epsilon}{3} \left(40 + \frac{72\mu^2}{r^8} \right) \right\} \right. \\ &\left. - \mathrm{e}^{\rho(r=r_{\max}) - \rho(r=r_{\max};\mu=0)} \int_0^{r_{\max}} dr r^3 \right) \left\{ 8 - \frac{80\epsilon}{3} \right\} \; . \end{aligned} \tag{2·32}$$

The factor $\mathrm{e}^{\rho(r=r_{\max}) - \rho(r=r_{\max};\mu=0)}$ is chosen so that the value of the proper length of the circle which corresponds to the period $\frac{1}{T}$ in Euclidean time at $r = r_{\max}$ for the two solutions coincide. Then we find

$$\mathcal{F} = -\frac{N^2 V_3 (\pi T)^4}{4\pi^2} \left(1 + \frac{3}{4N} \right) \; . \tag{2·33}$$

The entropy $\mathcal{S}$ and the mass (energy) E are given by

$$\begin{aligned} \mathcal{S} &= -\frac{d\mathcal{F}}{dT} = \frac{N^2 V_3 (\pi T)^4}{\pi^2 T} \left(1 + \frac{3}{4N} \right) \; , \\ E &= \mathcal{F} + T\mathcal{S} = \frac{3N^2 V_3 (\pi T)^4}{4\pi^2} \left(1 + \frac{3}{4N} \right) \; . \end{aligned} \tag{2·34}$$

Hence, as is expected, the presence of a non-trivial boundary does not influence the black hole thermodynamics,[12] which provides the corresponding description for dual SCFT at finite temperature in the next-to-leading order of the large-N expansion.

§3. Discussion

As we demonstrated with the example of the gravitational dual (HD gravity) of SCFT with two supersymmetries, the dynamical brane (the observable universe) may occur as the boundary of a d5 AdS black hole in the next-to-leading order of AdS/CFT correspondence. The coefficients of the surface counterterms are consistently fixed within AdS/CFT correspondence; they are not fine-tuned by the condition of the existence of the brane (which is the usual case in brane-world scenarios).

Moreover, the signs of coefficients (predicted by AdS/CFT) of HD gravity are such that the brane radius is larger than the horizon radius. In other words, the observable universe may be realized as the boundary of a multi-dimensional AdS black hole but outside of the horison. It could be interesting to further develop the details of such scenario.

Few remarks are in order. First, inside the horizon $r < r_h$, if we rename r as t and t as r, we obtain a metric corresponding to an AdS-like cosmological model:

$$ds^2 = -\mathrm{e}^{2\rho}dt^2 + \mathrm{e}^{-2\rho}dr^2 + r^2\sum_{i=1}^{d-1}\left(dx^i\right)^2 \ ,$$
$$\mathrm{e}^{-2\rho} = -\frac{1}{t^2}\left\{-\mu + \left(1+\frac{2}{3}\epsilon\right)t^4 + 2\epsilon\frac{\mu^2}{t^4}\right\} \ . \tag{3·1}$$

The leading-order behavior of the curvature here is the same as in the black hole case with $c = 0$, and we find that there is a curvature singularity at $t = 0$:

$$\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma} = 40 + \frac{72\mu^2}{t^8} \ . \tag{3·2}$$

Also there is a horizon at $t = r_h$. The singularity could be regarded as a kind of big bang. The topology of the spatial part is $S_1 \times R_3$ if we impose a periodic boundary condition on r and it is $R \times R_3$ if we do not. Here R_3 corresponds to the coordinates x^i.

We can consider the free energy analogue of $\mathcal{F}^{\cos}$ when $c = 0$ as follows:

$$T\mathcal{F}^{\cos} = \frac{8N^2V_3}{4\pi^2 T}\int_0^{r_h} dt\, t^3 = \frac{2N^2V_3\,(\pi T)^4}{4\pi^2 T} \ . \tag{3·3}$$

If $c \neq 0$, however, the free energy diverges, due to the singularity of $\hat{R}_{\mu\nu\xi\sigma}\hat{R}^{\mu\nu\xi\sigma}$ in (3·2) when $t \to 0$. This simple consideration suggestssome duality between black hole solutions and cosmological solutions in multidimensional HD gravity. As there exists a well-developed technique to study cosmological models, the above trick may be useful in the investigation of black hole interiors.

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